Appendix A: Simple-LP Gradient Design Formulation

To actually solve the gradient design problem using linear programming or SOCP, the matrix **A** and vector **b** are constructed using all of the constraints discussed in the text. For simple-LP formulation, the constraint matrix form is as follows:

$$
\left(\begin{array}{cccccccccc}
1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
-1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & -1 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & -1 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & -1 \\
\hline
C_1 & 0 & \cdots & 0 & 0 & S_1 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
C_P & 0 & \cdots & 0 & 0 & S_P & 0 & \cdots & 0 & 0 \\
0 & C_1 & \cdots & 0 & 0 & 0 & S_1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & C_P & \cdots & 0 & 0 & 0 & S_P & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & \cdots & 0 & C_1 & 0 & 0 & \cdots & 0 & S_1 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & C_P & 0 & 0 & \cdots & & S_P \\
\hline
\alpha C_1 & \beta C_1 & \cdots & 0 & 0 & \alpha S_1 & \beta S_1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\alpha C_P & \beta C_P & \cdots & 0 & 0 & \alpha S_P & \beta S_P & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & \cdots & \alpha C_1 & \beta C_1 & 0 & 0 & \cdots & \alpha S_1 & \beta S_1 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & \alpha C_P & \beta C_P & 0 & 0 & \cdots & \alpha S_P & \beta S_P \\
\hline
\gamma\tau/2\pi & \gamma\tau/2\pi & \cdots & \gamma\tau/2\pi & \gamma\tau/2\pi & 0 & 0 & \cdots & 0 & 0 \\
-\gamma\tau/2\pi & -\gamma\tau/2\pi & \cdots & -\gamma\tau/2\pi & -\gamma\tau/2\pi & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \gamma\tau/2\pi & \gamma\tau/2\pi & \cdots & \gamma\tau/2\pi & \gamma\tau/2\pi \\
0 & 0 & \cdots & 0 & 0 & -\gamma\tau/2\pi & -\gamma\tau/2\pi & \cdots & -\gamma\tau/2\pi & -\gamma\tau/2\pi
\end{array}\right)
\left(\begin{array}{c}
G_x[1] \\ G_x[2] \\ \vdots \\ G_x[N-1] \\ G_x[N] \\ G_y[1] \\ G_y[2] \\ \vdots \\ G_y[N-1] \\ G_y[N]
\end{array}\right)
\le
\left(\begin{array}{c}
\epsilon \\ \epsilon \\ \epsilon \\ \epsilon \\ \epsilon \\ \epsilon \\ \epsilon \\ \epsilon \\
\hline
G'_{\max} \\ \vdots \\ G'_{\max} \\ G'_{\max} \\ \vdots \\ G'_{\max} \\ \vdots \\ \vdots \\ G'_{\max} \\ \vdots \\ G'_{\max} \\
\hline
\psi' \\ \vdots \\ \psi' \\ \vdots \\ \vdots \\ \psi' \\ \vdots \\ \psi' \\
\hline
\Delta k_x + \epsilon \\ -\Delta k_x + \epsilon \\ \Delta k_y + \epsilon \\ -\Delta k_y + \epsilon
\end{array}\right)
$$

Horizontal lines separate four different types of constraints: boundary constraints, current constraints, voltage constraints and area constraints. Here $C_p = \cos(\theta_p)$ and S_p = sin

(θ_p) with θ_p defined as in Eq. 8. Also, $G_{\max}{}' = G_{\max}\cos\left(\frac{\pi}{P}\right)$ and $\psi' = \psi\cos\left(\frac{\pi}{P}\right)$. In the above expression, ϵ can have different values for different constraints. (As described in the text, γ is the gyromagnetic ratio, τ is the sampling period, and ψ is defined in Eq. 7.